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ABSTRACT

This report includes a general overview of the literature on computer use at the two-year college level and in specific academic disciplines, and a description of the development of a computer assisted instruction (CAI) program in linear algebra. The literature review was based on articles from journals, Dissertation Abstracts International, a computer search of ERIC, and inquiries to people using CAI. Materials are grouped by mathematical subject (pre-calculus, algebra, trigonometry, calculus, finite mathematics, and linear algebra) and computer uses for each are described in the areas of placement/diagnostic testing, drill and practice, demonstrations, tutorials, problem-solving, or simulations, with programs oriented towards drill and practice the most numerous. A rationale for the choice of topic for developing a CAI program points out the lack of programs available in linear algebra problem-solving, and the development of an interactive, goal-oriented, problem-solving program for linear algebra is documented. Programmed on a Compucolor II microcomputer, it demonstrates the linear transformations of rotation, dilation, and reflection on the two-dimensional plane in the form of a user challenge. Information given includes the underlying principles, a program overview, a modular flow chart, and specific user instructions (preliminaries, running the program, examples and explanations, and mathematical concepts). A 9-page bibliography is provided. (LMM)

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USE OF MICROCOMPUTERS AT THE JUNIOR COLLEGE LEVEL IN THE TEACHING OF MATHEMATICS

A Project Report
Presented to
the Faculty of
the Department of Mathematics
and Computer Science
San Jose State University

In Partial Fulfillment
of the Requirements for the Degree
Master of Science

by
Sheryl E. Edwards

August 1982

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This project is dedicated to my husband, John, for his patience, understanding, support, and encouragement throughout my studies.

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S.E.E.

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CHAPTER 1

INTRODUCTION

Computers are appearing in many colleges today. Services are available in over 90% of the institutions of higher education for computer use including instruction, research, and administration (Gillespie, 1981).

Computers touch almost every phase of our lives. They determine and address our bills, they regulate our electricity and process our phone calls, they can help our children learn, control our microwave ovens, and diagnose the ills of our automobiles.

Given the importance of the computer, it is shocking that a large proportion of our population knows so little about them. Perhaps this condition can be explained by the rapid rise of computer power; it has increased by a factor of ten every eight years since 1946! Computer literacy is gradually increasing. Introductory computer courses at colleges and universities are usually filled to capacity, and many high schools either have a computer of their own or use nearby computer facilities.

A 1981 report entitled COMPUTING AND HIGHER EDUCATION: AN ACCIDENTAL REVOLUTION (Gillespie, 1981) calls attention

to the importance of computing to national and educational goals and recommends actions to strengthen its role. He states that computers offer a major opportunity for increasing productivity. Examples include computer-aided design and manufacturing, managing information activities with terminals and data bases in businesses, and education and training through computer-assisted and computer-managed instruction.

In their 1972 booklet APPLIED MATHEMATICS IN THE UNDERGRADUATE CURRICULUM, the Committee on the Undergraduate Program in Mathematics strongly recommended the incorporation of both the computer and realistic applications into classes in applied mathematics (Caton and Grossman, 1980).

In 1980 the National Council of Teachers of Mathematics recommended that mathematics programs take full advantage of the power of computers at all grade levels (NCTM, 1980).

In the educational setting, the computer can be used as a "tutor" in which text is presented in some programmed fashion. This mode of using the computer in education has its own values and utility, but may not be the best method to which microcomputers are suited. Students at the computer may proceed at their own rate, interact with the computer, and be addressed by name. Slower learners may get extra help and practice, while faster learners may proceed far beyond the minimum scope of the course. Many

slow learners and unmotivated students develop into successful scholars when they are permitted to work on interesting projects in a computer-enhanced learning environment (Garrett, 1979).

Interaction, immediate feedback and reinforcement, independent pacing, and the computer's endless patience all contribute to the uniqueness of computer use in education. Providing alternative learning paths, recordkeeping, and evaluating performance may also be involved.

In addition to presenting text and asking questions of the student the computer can make graphics presentations, can check computations based on input, and present problems and exercises using the material.

The computer can interact with each student and help recognize and develop competence in weak areas. The computer has the ability to perform difficult computations and plot graphs quickly so as to reduce the tediousness involved in some problems. Students may become more involved in the learning process through use of materials which ask the user to make appropriate selections towards goal-oriented applications using the information given.

When I undertook this project, I wanted to investigate how computers are used in the teaching of mathematics at the junior college level. Having seen very few programs available at this level and having seen a minimal amount of literature on the topic, my first task was to find out

exactly what was being done.

I gathered articles on the subject by researching the AMERICAN MATHEMATICAL MONTHLY, THE MATHEMATICS TEACHER, ERIC catalogs, DOCTORAL DISSERTATION ABSTRACTS, and initiating a computer search of ERIC listings. Other publications which proved to be most useful were the NATIONAL CONSORTIUM ON USES OF COMPUTERS IN MATHEMATICAL SCIENCES EDUCATION NEWSLETTER and PROCEEDINGS from the Computers in Education Conference at Arizona State University, 1981. Letters were also written to various people for more information on use.

After gathering articles on the field of interest, I divided them into categories by mathematical subject. As I wrote, I grouped information into subcategories according to whether the uses were in the areas of placement/diagnostic testing, drill and practice, demonstrations, tutorials, problem solving, or simulations.

Upon completion of the review of literature I discovered that some material is available in the areas of pre-calculus, algebra and trigonometry, finite mathematics, and calculus. Programs oriented towards drill and practice were the most numerous. Chapter Two, Sections one through four, describes the information found in these areas.

Linear algebra seems to be an area where computer use is not very well represented. Hence, I decided to more thoroughly study the use of computers in linear algebra, especially emphasizing the use of programs other than drill

and practice because of the small amount of material found of this type. Chapter Two, Section five, describes the information found in this area.

After a more thorough review of computer-oriented materials for linear algebra, I decided to develop a progra for goal-oriented demonstrations of linear transformations in the plane. Chapter Three describes the reasons for this choice. Chapter Four describes the methodology used in developing the program with conclusions given in Chapter Five.

CHAPTER 2

REVIEW OF LITERATURE

Section 1. Pre-Calculus Courses

Because the California Community College system is open to any student over 18, students exhibit a wide range of abilities. There are great numbers of students enrolled in classes in arithmetic and algebra. These students may have forgotten everything, or failed to understand anything, or simply never had anything to do with mathematics before. In this group is a wide variety of students who are suddenly required to study a subject which they may have been fleeing for years. Computers can help these students by administering objective placement tests for the topics of arithmetic through trigonometry, scoring these tests, keeping records of student performance, as well as to provide individualized mathematics skills instruction. The correct placement of these students is critical to insure that each student has maximum opportunity to succeed (Stein, 1972).

The abilities and diverse backgrounds of community college students contribute to the need for remedial as well

as developmental mathematics courses, which involve students who have mathematical competencies less than junior high school algebra. The computer can be used to determine course placement; that is, of the courses required to be taken, computer administered and scored placement tests determine the courses of which the student shows mastery. Computers can be used for diagnostic testing; that is, of the work required to be done on a topic, what does the student already know well?, on what does he need review?, and on what does he need instruction? Computers can be used to administer achievement tests at the end of an entire course. The computer can also be used to provide error analysis for student solutions to exercises.

Computers have been used to determine placement in college mathematics courses. Riggle (1975) uses computers to determine course placement in the developmental mathematics program at Cuyahoga Community College in Ohio. Miami-Dade Community College has a computer assisted program of assessment and placement for arithmetic through trigonometry in its Mathematics Learning Center (Palow, 1978, 1979). Mohawk Valley Community College has available a placement test for Technical Mathematics (Brooks, 1981).

Fuhrer (1973) at New York City Community College found that computer assisted testing which employs hints and feedback to examinees, and allows for multiple attempts on each item with weighted scoring procedures is a better

predictor of success in college technical mathematics than standardized multiple choice testing; and that students considered it more fair than standardized testing.

Diagnostic testing is another area where computers can be useful. Hirmanpour (1980) wrote computer programs for diagnostic testing in college remedial mathematics. Administration of tests and a record of each student's test performance were also handled by the computer. Olivier (1973) uses the computer facilities of the Ontario Institute for Studies in Education and the National Research Council of Canada for on-line diagnosis and instruction of mathematics skills. DeKalb Community College, Georgia, has diagnostic/placement exams for their remedial courses (Rice, 1981).

Computers are used to administer achievement tests at the end of a course. DeKalb Community College, Georgia, has departmental exams for remedial courses on computers (Rice, 1981). Miami-Dade Community College also has final exams on computers as a part of their Directed Independent Study (DIS) program which provides an alternative to classroom instruction in arithmetic, introduction to algebra, college algebra, and trigonometry (Palow, 1978, 1979).

Educators at San Diego State University have constructed a computerized mathematics testbank covering college algebra, trigonometry, and intermediate algebra

(CATIA). The computer is used for the generation and scoring of tests. CATIA, which makes approximately 4000 questions available, has been used for placement, testing, retesting, testing in the regular classroom lecture format, and producing final group examinations (Overton, 1981).

In the area of individualized mathematics skills instruction, computers are often used for computer-assisted instruction (CAI). CAI is concerned with learning processes that are related to instruction and may include these categories of application: drill, practice, demonstration, and tutorials. Drill involves repetition of facts, definitions, theorems or formula which are to be committed to memory; practice includes application of principles to exercises in which they must be applied; and demonstration involves computer created situations, the results of which students can cause to change by modifying initial data; tutorials include any combination of the previous three categories (Fitting, 1980).

CAI lessons can serve as text and tutor while compelling students to be active participants in their own learning. Depending on the sophistication of the lessons the computer can provide assessment or interactive remedial instruction. Students work at their own pace while their CAI lesson monitors their progress and commonly prevents them from continuing to more advanced instruction unless mastery is demonstrated. Students are kept informed of

their progress through immediate feedback and achievement summaries. Students have varying amounts of control over their learning in that they can review previous instruction, request special help, or continue on to enrichment activities (Magidson, 1978, and DeLaurentiis, 1980).

There are many examples of the use of computers in CAI mode. Flower and Craft (1981) use CAI for a non-credit review of high school algebra, geometry, and trigonometry using the unique ability of the CAI system to concentrate only on those areas needed by the student thus allowing for more individualized lessons. A teacher is also available for questions and help in areas where more than just a review is necessary. The availability of this class as an elective helps to alleviate high enrollment and dropout rates in engineering classes, thus providing somewhat of a screening process.

Computers are used in Suffolk Community College's remedial mathematics courses primarily for CAI. Students proceed at their own pace, a particularly valuable asset in courses like remedial arithmetic where the students' backgrounds vary widely. This is a major advantage for students who require only a brief review. At the other extreme are students who need literally hours of repetitive, randomly generated problems on each phase of the course. For these students programs are available on

topics such as operations with fractions, operations with decimals, factoring, and combining like terms. These CAI programs also store the results for each student and so provide a permanent record for the instructor of which student used which program on which date and how successful the student was (Gordon, 1982).

DeKalb Community College has computer modules for remedial algebra students (Rice, 1981). Computers are used to administer modular units of individualized instruction which are accompanied by regular lectures in the developmental mathematics program at Cuyahoga Community College in Ohio (Riggle, 1975). At Grinnell College (1981) a series of interactive programs have been developed for students to use in pre-calculus mathematics. New York City Technical College uses computers for CAI in their mathematics learning laboratory (Africk, 1981). Marymount Manhattan College uses computers as an instructional aid in class and as a tutorial aid in their mathematics lab. They have programs suitable for developmental mathematics, pre-calculus, and more (Murray, 1981). Lane Community College in Oregon has several programs students can use on elementary differentiation, graphing trigonometric functions, and random number illustration by graphing (Miller, 1981). They also have several individualized, independent-study courses in their Mathematics Resource Center (Fast, 1980).

Computers have been used successfully for analysis of student errors. Travis (1978) showed that the computer can identify and rectify student errors in the multiplication process. Thus showing that computers can be used to diagnose and remediate learning difficulties.

There are also available one-hundred-six lessons from the PLATO (Programmed Logic for Automated Teaching Operations) computer project for community colleges and adult education in topics ranging from elementary arithmetic to function theory and trigonometry. These programs are operable on Control Data Corporation computers only (DiBello, et. al., 1977, in catalog form).

An extension of CAI is computer-managed instruction (CMI) which uses the computer to assist in classroom management. This function can include giving pretests to students to determine their readiness to learn new topics, giving posttests to measure learning, evaluating students' answers to test questions to discover error patterns, keeping academic records on each student in a class, and prescribing appropriate materials and methods for instruction based on student progress through a course of study. The computer can monitor the learning sequence and provide students with formative evaluations and records of their progress. By testing, evaluating, and prescribing tasks for students, CMI systems allow for the easy flow of students through programs designed to meet individual

learning needs, and handles some of the routine chores of teachers which may give them time to work with individual students in diagnosing and resolving more subtle learning difficulties (Caldwell, 1980, and Garrett, 1979).

CMI is used at Miami-Dade Community College to augment its Mathematics Learning Center with a computer-assisted program of individualized instruction. The instruction program includes: a system of assessment and placement for arithmetic through trigonometry, determines a student's preferred way of learning through an inventory test, an instructional system to provide students with a sequential series of learning units with computer-graded tests at the end of each unit, and a computerized system of evaluation and management. One component of the program is the Directed Independent Study (DIS) program which provides an alternative to classroom instruction in arithmetic, introduction to algebra, college algebra, and trigonometry. The DIS program consists of twelve performance-based units of instruction (Palow, 1978, 1979).

Miami-Dade Community College's Response System with Variable Prescriptions is used to assist in individualized instruction and record-keeping. The computer can print feedback reports to students, with test scores and remarks to negligent students. It provides instructors with reports on student performance and collective class data. The computer will also assign sets of personalized

exercises to students, focusing on a skill used incorrectly on a test (Emerson, 1978).

Egan and Verno (1976) at West Chester State College in Pennsylvania, developed an Individualized Learning System course for college freshmen, designed to satisfy a general education requirement in mathematics. Students respond to the computer, and the computer takes care of many of the routine classroom management chores. Laffey (1981) uses a Computer-Managed Education system in Seattle Community College's mathematics lab to provide course material, and to facillitate the record-keeping and decision-making involved in instruction.

Since there are definite and significant individual differences among all learners, computer technology provides an excellent medium for assisting students of differing abilities in learning. CAI offers a means of providing instruction in drill and practice of facts and skills that so many adults have avoided most of their life. With CAI students can study subjects through a computer-centered learning system where a computer terminal presents interactive information. Use of computers in a mathematics laboratory for remediation of mathematics skills provides for more drill and practice than the student might receive in a large classroom situation. Here, the ability of the computer to lock in and intensify instruction for each student's weaker topics may be used to a great advantage.

The computer can be effective in the generation of problem sets and examinations, the monitoring and tracking of student progress, the assessing of student abilities, and the placing of the student in the appropriate learning situation. The computer is capable of providing educators with technological assistance in several important and time consuming tasks. Teachers can use CMI systems to score tests, to keep detailed records on the grades and progress of individual students, as well as to diagnose learning difficulties. It can even recommend how teachers might group students for certain instructional activities based on the prerequisites students have met. CMI can help teachers in their planning and help cut down on clerical time. The system can help teachers individualize, and with detailed records on individual performance always available, teachers can see exactly where a student needs help.

Section 2. Algebra and Trigonometry

In the fields of algebra and trigonometry computer programs support single lessons, full topic modules, or entire course reviews. Computer programs provide drill, practice, demonstrations, tutorials, experiences in problem solving, and simulations. Many of these types of programs are used in trigonometry classes at San Jose State University (SJSU).

Drill is a repetition of questions the answers to which require memorized facts, definitions, theorems or formula. The student is provided with, for example, part of a formula, and asked to type in the balance. Since the basic trigonometric identities need to be memorized for efficient application, drill is appropriate. In a SJSU program the drill gives one member of a trigonometric identity, and the user chooses the matching member from a given list (Fitting, 1980).

Practice involves the application of principles in various exercises. A computer program involving practice may provide suggestions on the principle and how to use it whenever exercises are done incorrectly. Caldwell and Polley (1975) designed computer-assisted practice which analyzed students' answers to computational exercises dealing with quadratic equations and equations of straight lines. The student, seated at a teletype, receives an

exercise with randomly chosen parameters. The student then solves the exercise and enters his solution. If the solution is correct, another exercise is presented. If the solution is incorrect, the computer then works the exercise incorrectly in several different ways, each time checking its answer against the student's. If a match is obtained, the computer suggests that the student has made a certain error and asks him to try again. If none of the incorrect answers match the student's answer, then the computer will check the student's work step by step. For example, in the case of the quadratic equation program, the student is first asked "What did you get for A, B, C?" and then later "What did you get for B^2 - 4AC?" Finally, if the student does not enter the correct answer, he receives a detailed solution. If a student requires detailed solutions to several problems, he is asked to contact the instructor. This approach offers immediate feedback which might not be available if he had to compete with other students for help from the instructor.

Another example of practice is the program on radian measure in the trigonometry package used by SJSU. The program diagrams a unit circle, then randomly chooses a number and changes the color of an arc, whose central angle has this measure in radians. The user is to determine how the length of this arc compares to the radius and hence how many radians are in the central angle of this arc. If the

answer guessed is incorrect, the correct answer is suggested, and the user must type and enter this answer to proceed (Fitting, 1980). This program could be improved by reminding the user that he is to provide the answer to the nearest tenth, and by providing suggestions which will lead the student closer to the correct answer without actually giving it to him.

Demonstration involves computer created situations, the results of which students can cause to change by modifying initial data. As an example of demonstration, the mathematics lab at Albany Junior College has several computer programs available for computer-assisted discovery learning of selected trigonometry units to supplement regular classroom instruction. They include cause and effect relationships between particular parts of equations and their graphs, sketching graphs from equations, determining equations from their graphs, and an introduction to polar coordinates. The programs include a demonstration program on the solution of a system of linear equations, and demonstration plotting routines on polynomial equations, trigonometric equations and polar equations which allow the user to change the highest power of x and constants (Kelly, 1980).

In a discovery oriented demonstration of the concept of trigonometric identity used at SJSU, the student is asked to specify an angle in radians. The computer then displays the

values of the sine and cosine of each angle entered and the squares of each of these functions. It next requests the user to add the values of the squares of the sine and cosine functions, and to enter the sum. The user may repeat the entry of ten values as many times as necessary to determine the truth of the pattern discovered. This program involves discovery of three identities and presents an interactive summary of the material (Fitting, 1980).

Shneiderman (1974) used the computer's graphic capabilities and computational speed to consider a question often overlooked in algebra due to the tediousness of the task. He programmed the computer to plot a polynomial, given the coefficients. This eliminates the drudgery of drawing the graphs of many polynomials, and allows investigation of the effect the coefficients of polynomials have on the shape of the graph. Related problems include giving the student a graph of a polynomial and challenging him to alter the coefficients to find an equation which fits the graph, and also to form a polynomial to go through certain points of the x-axis, thus developing a sense of interaction between the coefficients and the roots of polynomials. Another example of demonstration is the program entitled "Graphs of Sine Functions" in the trigonometry package used by SJSU. The program graphs the sine function and gives the user the opportunity to vary A, B, and C in the equation $y = A \sin(B\ (x + C))$. The

computer graphs each in a different color on the same axes, allowing the user to see what effect each has on the appearance of the wave (Fitting, 1980).

Tutorials involve the presentation of new knowledge combined with evaluations of how well a student is acquiring the knowledge. The evaluations may be used to route the student through a branching program to provide review, practice, or drill and reinforcement. In one form the tutorial is a program in which the student will read some material and answer a question (or two) about the reading; the computer will record the number of answers correct and incorrect. In more imaginative forms the tutorial helps the student to discover the principles to be learned and provides adequate application of the principles. An example of tutorials is a package of programs by Howard Jensen (1980). This package, which includes the student's name in feedback, is on the properties of slope, midpoints, distance, and equations of lines. These programs provide explanations and graphics demonstrations, ask questions at each point for reinforcement, and provide practice with the concepts. The computer is programmed to ask the user to see the instructor if he solves too many problems incorrectly. A coordinate quiz is also a part of the program package. In the quiz, exercises are presented, which the user works on paper, then enters his solution. The computer then gives the student his score on the quiz.

In problem solving the use of computers can involve computational procedures which allow students to gather or operate on data the results of which will lead them to the solution of the problem at hand. Olson (1979), in problem solving which involved looking for patterns, found a calculator to be sufficient to perform the computational tasks involved in testing out hypotheses by trying various numerical examples. For example, does the following generate a pattern: $5^2 - 5 = 4^2 + 4$, $6^2 - 6 = 5^2 + 5$? The students can generate examples of the type $1776^2 - 1776 = 1775^2 + 1775$, and by using the calculator to test other examples, the students may conjecture that if $a - b = 1$, then $a^2 - a = b^2 + b$. The hypothesis then can be proved algebraically.

Snover and Spikell (1978) posed a more complex problem, for which a computer was better suited. Noticing that $153 = 1^3 + 5^3 + 3^3$, that is, that 153 is equal to the sum of the cubes of its digits, the student, with the help of the computer, could search for other numbers with the same property. Once found, the student could compare and conjecture regarding the form these numbers take.

Gabai (1976) used computers to motivate an exercise in graphing where students construct "graphcodes" which are sets of equations, defined over specific intervals, for which the corresponding graphs resemble letters and words. The computer plots the student entered graphcodes. This

provides an enjoyable exercise in graph sketching and a greater motivation to understand the relationships between an equation and its graph.

Computers are also used for simulation, which involves presentation of a problem oriented environment in which the student makes irrevocable decisions and suffers the consequences, as a supplement to algebra. Mathers (1976) used BASIC to simulate Queuing Theory at the barber shop; and, Dunn and Wright (1977) used BASIC to simulate various aspects of the economy of the United States by means of projections into the future, to augment algebra classes with examples relevant to the real world, and examples of applications of mathematics outside the physical sciences.

At Grinell College (1981) a series of programs have been developed to help students improve their algebraic and geometric intuition and technique. These programs are a combination of drill and practice, testing, dialog, and simulation. The programs are in two parts. The first is "Speed up your Algebra" with error analysis by the computer. The second is "Test your Skills" on solving linear equations, completing the square, simplifying exponential expressions, and some geometric topics, such as "Match the Graphs" which help the student to understand the relations between parts of an equation, for example coefficients and exponents, and significant features of a graph. Programs are available on pth power functions with

any "vertex", any leading coefficient and any power, and on sine functions with any mean, amplitude, frequency and phase shift.

Soper (1981) uses computers to provide support for review of elementary algebra. Included are drill in signed numbers and factoring quadratic trinomials. These programs give the student a choice of level of difficulty, then make up problems using a random number generator so the student can practice as long as he likes without exhausting the supply of problems. In these programs the student is immediately told whether he is right or wrong and no record of student performance is kept. Pfetzing (1972) used the computer to present the concepts of linear inequalities, absolute value, and functions. Brooks (1981) at Mohawk Valley Community College has BASIC programs for adding and subtracting negative numbers, distance and solution problems, factoring trinomials, graphing elementary functions, and for solving linear equations. At New York City Technical College available programs for algebra include factoring trinomials, using the quadratic formula, and simplifying radicals (Africk, 1981). Lane Community College in Oregon has programs on trigonometric function graphing (Miller, 1981).

Flower and Craft (1981), and Palow (1978 and 1979) used computers to review and present high school algebra and trigonometry topics. Rice (1981) uses computer modules for

remedial algebra students, and Rockhill (1981) at the SUNY Brockport Microcomputer Project has developed instructional modules for mathematics courses including college algebra.

Blattner (1981) has written demonstration programs to do such things as sum several thousand terms of a series, plot graphs, and provide probability simulations. Steen (1981) at St. Olaf College in Minnesota uses computers and a library of graphics programs and subroutines to permit students to display two and three-dimensional graphs, as well as instructional units for pre-calculus mathematics which include extensive simulation coordinated with a detailed written study guide that enables use of the computer as a source of data, of conjectures, and of patterns. The Georgia Institute of Technology utilizes the graphical and numerical capabilities of the computer for the illustration of key ideas in their mathematics classes. The catalog of programs they have includes such routines as: function plotters; functions and integral plotters; parabolas; quadratic surfaces; trigonometric functions; harmonic oscillation, and more (Christensen, 1981).

In algebra and trigonometry the computer is used frequently to assist instruction. Computers can be used to present drill, to practice with hints on the principles involved, for discovery learning or reinforcement of concepts by demonstration, for tutorials involving the previous three categories, to provide opportunity for

problem solving involving computations suitable for a computer, and for simulation of realistic problems and situations.

The computer seems to provide interactional and motivational benefits, as well as other instructional advantages. One is the computer's ability to analyze student's answers to computational exercises during practice, and provide immediate, individualized help. Another is to allow implementation of questions previously omitted from the classroom due to the tediousness of calculations involved. The computer's computational speed can be used to search more effectively than student, instructor or text for number patterns. A computer can be used to generate numerous graphs for illustration of concepts, and it can also generate data which students can examine and base conjectures on, which may then be proved or disproved.

Section 3. Finite Mathematics

In finite mathematics, computers are used in the tutorial instructional mode.

At the SUNY Brockport Microcomputer Project, tutorial modules have been developed for finite mathematics on topics such as matrices, linear equations, and solving linear programming problems (Rockhill, 1981). Wyzkowski (1981) uses instructor-written and library programs in BASIC for the inverse of a matrix, and linear programming in finite mathematics classes. These programs are also used to introduce students to some of the capabilities of a computer and to alleviate some of the tedious calculations, after the students have attained an understanding of the algorithm by doing some easier problems without using the computer program.

At the Memorial University of Newfoundland, a course in finite mathematics is offered for students who do not wish to study calculus. A collection of supplementary programs for this course are available, with remedial intent. After pressing a key, a particular exercise appears on top of the screen. The student is asked to respond to a sequence of questions which lead to a complete solution of the exercise. The responses entered are compared against stored ones. "Help" is available at various stages. At the end of an exercise, the cumulative number of errors made is

given, with hope that this feature will allow the student to monitor his progress. The material consists of a synopsis and exercises in logic, set theory, counting theory, probability theory, and mathematics of finance (Lal, 1981).

Section 4. Calculus

In calculus courses computers are used in a variety of ways. They may be used as an auxiliary tool to provide printed graphics displaying functions and limits of functions. They may be used for solving problems which involve computational procedures; these may be direct solutions or allow students to gather or operate on data and arrive at results which lead them to the solution of the problem. Since many computations can be performed quickly, computers can be used to provide an approach to calculus from the discrete point of view. Computers can also be the main ingredient in a course concurrent to the regular calculus sequence which emphasizes computational techniques, as well as being used to present tutorials and instructional modules to supplement regular calculus courses.

There are many instances where computers are used to display graphics as an aid to calculus. Myers (1981) has programs for use in calculus on sketching curves defined parametrically, the area bounded by a curve in polar coordinates, and the volume of a solid of revolution, as well as graphics to provide curves and surfaces in three-space. One of his packages includes images of surfaces, a program for use in generating additional surfaces, and a program which provides user-controlled

animation (rotating, scaling) of images of surfaces. These programs are intended for instructor use as a lecture aid.

Shneiderman (1974) uses the computer and a light pen in an interactive computer graphics system to make it easy to manipulate first to fourth degree polynomials with student chosen coefficients. The derivative of the polynomial can also be shown on the same set of axes at a lower intensity.

Georgia Institute of Technology uses graphics generation devices for the illustration of key ideas in their mathematics classes. They have a catalog of programs including function plotters, quadratic surfaces, and convergence of upper and lower approximations to the definite integral (Christensen, 1981).

The Memorial University of Newfoundland has demonstration programs for calculus which draw the graph of a function and find zeros, maximums, and minimums over a given interval (Lal, 1981). Eisenberg (1981), at the University of Massachusetts, uses computers for in-class demonstrations on the concept of the definite integral as the limit of Riemann sums. Blattner (1981) uses computers for in-class demonstrations, utilizing programs that perform numerical integration, illustrate the derivative, and plot functions, polar, and parametric equations. Rhode Island College uses computers for demonstration in the classroom for their calculus courses (Sedlock, 1981).

Long (1976) used computer graphics to produce a

sequence of computer generated pictures entitled "Peaks, Ridge, Passes, Valley and Pits--A Slide Study of $f(x,y)=Ax^2 + By^2$." He uses these pictures to present concepts in the study of functions of two variables in which a regular non-planar point on a smooth surface can be classified as one of three distinct types: elliptic, parabolic, or hyperbolic. Elliptic if the product of A and B is positive, parabolic if the product of A and B is zero (A not equal to B), or hyperbolic if the product of A and B is negative (planar if A=B=0).

Johnsonbaugh (1976) used the computer to calculate partial sums of an infinite series to illustrate limits. Students then examined the resulting partial sums to determine the convergence or divergence of the series. Flynn (1974) presents two methods which will approximate a definite integral, and may be programmed for the computer. The trapezoidal rule gives the student an idea of what an integral represents, and how to approximate it. The Monte Carlo method uses random numbers generated by the computer and provides a good introduction to probability theory.

At SUNY at Cortland, N. Y., the computing facility includes an x-y plotter which allows simultaneous computing and graphing of upper and lower sums, to aid the student in developing an intuitive understanding of the definite integral (O'Loughlin, 1976).

In calculus courses at Simmons College canned computer

programs are used for problem solving. Canned programs take up less class time and homework time than student written programs since the student does not need to know how to write programs. Typical programs investigate the limit as x goes to zero of (sin x)/x, estimate f(xo + h) by the tangent at xo for such functions as f(x)=x^2, $\sqrt{x}$, or sin x, and Riemann sums. The computer programs were written to be self-contained, and to make the student feel as though he were in control of routine calculations. The programs were written to "talk to" the student, and comments were used liberally so the student could follow the calculations involved. The idea behind the development of these canned programs was to show enough of the computations for the student to feel that each step was a simple arithmetic operation he was quite capable of doing by hand. As a consequence students were more apt to believe such concepts as limits and convergence, and the calculus seemed more concrete. Students also felt that in estimating pi and constructing logarithmic and trigonometric tables they were being let in on some things that had always been held secret (Menzin, 1976).

In a program which computed upper and lower sums for the area under y=1/x, 1<=x<=2, a comment before the calculation began reminded the student that this was a monotonic decreasing function and therefore the maximum and minimum values could be found at the left and right hand

endpoints of any subinterval. Then the computer gave the formula x(i)=1 + i/N (student chosen N) and filled in a table with the headings: i, x(i-1), x(i), y-min, y-max, inscribed area, and circumscribed area. Finally the terminal added up the inscribed and circumscribed areas to produce the upper and lower sums, announcing it was doing exactly that (Menzin, 1976).

At Simmons College students used the extended mean value theorem (truncated Taylor series), the trapezoidal rule, and Simpson's rule to construct trigonometric and logarithmic tables. They also compared the error estimates of these methods for various numbers of subdivisions. Students used the inscribed and circumscribed area under $x^2 + y^2 = 4$, $x>=0$, $y>=0$ to estimate pi, used Newton's Method to estimate the root of an equation, and investigated the limiting processes, both convergent and divergent (Menzin, 1976).

In a report by O'Loughlin (1976), the results of a study conducted with two calculus classes were discussed. One class was taught by the traditional lecture method and the other used lecture and mini-computer. The computer enabled the student to evaluate a number of commonly used mathematical functions such as ln x, e^x, and sin x. Its use allowed discussion of numerous examples of a concept, inclusion of topics normally omitted or reserved for a subsequent course, and inclusion of more realistic

applications of the calculus. Additional topics on limits of functions, applications of the derivative, and numerical integration were also covered in the lecture and mini-computer class. It was found that the class which used the mini-computer performed as well on all concepts as the other class, but did significantly better on topics involving a function and its derivatives, the definite integral, and solving maximum-minimum problems. This study has shown that by using the mini-computer, the student can work on enrichment topics and topics not normally considered without weakening the topics usually covered.

Optional courses which may be taken concurrently with the regular calculus sequence can give students practice in exercises on the computer. At Duke University, in freshman calculus, two to three sections per semester (out of thirty) meet an extra hour to do computer-related exercises. In sophomore calculus, one section (out of ten or twelve) also meets to work on computer exercises (Smith, 1981).

Hethcote and Schaeffer (1972) use computers in a one semester hour course called "Computational Techniques of Calculus" at the University of Iowa. Problems from this list are assigned to be done on computers available in their computer laboratory:

1. Compute tables of given functions (polynomials, discontinuous, rational,

trigonometric, and exponential functions) and graph them.

2. Compute tables of given functions, derivatives and antiderivatives, graph them, illustrate maximum points, increasing functions, areas, etc.

3. Find limits experimentally.

4. Evaluate e to four significant figures by using its limit definition.

5. Evaluate pi to five places by using the perimeters of a sequence of regular polygons inscribed in a circle.

6. Find derivatives experimentally by evaluating limits of difference quotients. Verify the chain rule.

7. Write a program to find the maximum and minimum points of any cubic polynomial on a given interval.

8. Find zeros of a given equation using Newton's Method.

9. Find approximate values of integrals by using the Riemann sum and by using the trapezoid rule.

10. Compute tables of values and graph a given function and various approximations by truncated Taylor series on the same set of axes.

11. Verify L'Hopital's rule by computing the limits experimentally.

12. Evaluate the sums of infinite series experimentally. Compute the approximate values of a function by using its Taylor series.

Gordon (1979) uses computers in "A Discrete Approach to Computer-Oriented Calculus" at Suffolk Community College in their calculus sequence and in an applied calculus course for business majors at C. W. Post College. The discrete approach uses results from the calculus of finite differences and finite sums which leads to applications and significant tools to deal with them. Most topics are first developed in the discrete setting and then extended to the continuous case by applying the limit. This provides a constant review of all topics, a definite aid to most students. The approach provides the context for developing numerical algorithms for approximating functions and finding where all the "given" functions come from. It demonstrates the use of discrete as well as continuous models, especially in nonphysical sciences where processes are inherently discrete rather than continuous. The discrete approach provides an opportunity to introduce discrete approximations into calculus courses in a simple, yet natural way. It supplies a means for developing mathematics appropriate to an applied mathematician's, scientist's, or engineer's point of view, so students will have an idea of how nonlinear or extremely complex situations can be treated. This approach

allows development of approximation and interpolation methods such as Lagrange's formula and Newton's formula. These developments give the student a feeling that this material constitutes an integral part of the subject, and also provides an opportunity to use the computer in solving exercises which would have been far too complicated to consider otherwise. For example, it is easy to introduce the idea of fitting a polynomial graph to given discrete data. Other techniques for enriching a calculus course become accessible, one involving the numerical approach to derivatives.

Suffolk Community College has two groups of programs for calculus, one primarily computational in nature, the other purely graphical. Among the numerical programs, for instance, is one that will analyze any given function to determine intervals in which it increases or decreases and will determine the approximate location of its critical points. All the user has to provide is the function, the interval [A,B], and a step-size Δx. Another program dramatically portrays the delta-epsilon definition for limits. For any choice of function, limit point A, and presumed limit L, the computer will calculate a value for delta (if it exists) for any given epsilon. In this way, the student gains an emotional conviction in addition to just an intellectual one that a delta will exist for any epsilon when the function is continuous. There are also

programs that will graph virtually any continuous function given in terms of x, in terms of a parameter t, or in terms of a polar coordinate theta. All that need be supplied is the function itself and an interval of definition; the resulting graph is produced in about ten seconds (Gordon, 1982).

Garfunkel (1972) uses computers in a laboratory and computer-based first-year calculus course. The laboratory reintroduces the history of the subject and the sense of discovery. Students formulate and test models of physical, chemical, and biological systems. The use of the computer allows students to work with less idealized, more realistic situations. The course consists of ten units designed to cover the equivalent of the first-year calculus course at MIT. There are fifteen additional units leading into more advanced areas such as series, differential equations, and multi-variable calculus. The main features of this course are the laboratory, the computer, the individualization, and self-paced instruction.

Computers have been used to present instructional modules as well as to provide tutorial assistance. Mohawk Valley Community College and the SUNY Brockport Microcomputer Project both have instructional modules for the calculus (Brooks, 1981, and Rockhill, 1981). Marymount Manhattan College uses computers for the calculus as an instructional aid in class as well as for a tutorial aid in

their mathematics lab (Murray, 1981). Wyzkowski (1981) uses instructor-written programs to give students immediate feedback on the definition of the derivative, the definition of the definite integral, and the trapezoidal rule in calculus.

Thesing and Wood (1974) use the computer as a discovery tool in calculus for the estimation of the limit of a function at a point and the derivative and antiderivative functions associated with a given function. They use the computer in a laboratory designed to allow for student discovery and experimentation with guidance and programming help provided so that student efforts are directed toward successful conclusions.

At Albion College in Michigan, calculus students run programs from COMPUTER ORIENTED CALCULUS by Sheldon Gordon. Programs for integration using Riemann sums and for plotting curves in polar coordinates were also used in the classroom (Greenwell, 1981).

The ideas in the calculus can be made more concrete when computers are used as a supplement to the sequence. Computers may be used as an auxiliary tool to provide graphics demonstrations. They can be used for solving problems involving computational procedures. They may be used as the main ingredient in a course which emphasizes computational techniques, as well as being used to present tutorials and instructional modules. Computers can be used

to allow discussion of many examples of a concept, for inclusion of topics normally omitted or reserved for a subsequent course, and for more realistic applications of the calculus. The intuitive ideas of limits, integrals, and derivatives can be enhanced when using computers to supplement the calculus sequence.

Section 5. Linear Algebra

Implementations using computers in linear algebra generally involve one of three instructional modes. In the first, students program solutions to typical linear algebra exercises. In the second, the computer provides direct solutions to exercises which the student enters while running a prewritten program. The third technique involves computer demonstrations of matrix manipulations and transformations.

Eisenberg (1981) uses computers as a part of a linear algebra course for mathematics/statistics, computer science, and some other science majors. He teaches APL and assigns APL programming projects such as Gauss-Jordan row reduction and orthonormalization by the Gram-Schmidt process.

Hethcote and Schaeffer (1972) use computers in a one semester hour course "Computational Techniques of Linear Algebra" at the University of Iowa. Problems from this list are assigned to be done on computers available in their computer laboratory:

1. Write a program to do vector and matrix arithmetic.
2. Determine if a vector is in the spanning set of a set of vectors, and find the coefficients in the dependence relation if it is.

3. Write a program to do matrix inversion by row elimination.

4. Calculate determinants by elimination.

5. Solve a system of linear equations by iteration.

6. Find eigenvalues and eigenvectors by the power method.

Linear algebra students at Walla Walla College are expected to do some of their homework with the computer and write their own programs (Soper, 1981).

At Duke University one linear algebra class (out of six or seven) use computers (Smith, 1981).

In a different light, England (1978) uses APL notation in linear algebra. The students' use of APL notation is designed to help in the representation of concepts and ideas involved in linear algebra and helps in their analysis and exploration.

At the Memorial University of Newfoundland, programs were written to supplement linear algebra in two packages. One to solve exercises involving vectors in R^3 , and the other to solve linear equations by various methods (Lal, 1981).

There are also several computer programming texts written as supplementary material for linear algebra courses in which the programs are primarily of

computational nature such as A COMPUTER SUPPLEMENT TO LINEAR ALGEBRA: BASIC by Caton and Grossman, and COMPUTER APPLICATIONS IN FINITE MATHEMATICS AND CALCULUS by Ziegler.

Ziegler (1981) states that the purpose of his book is to provide students with an opportunity to solve problems that are too complex or time-consuming to be done by hand. The computer is implemented as a tool to be used in the problem solving process, enabling students to deal with more realistic and meaningful applications of the mathematics they are studying. No previous computer programming experience is required. Programs presented for linear algebra include solution of a system of linear equations by Gaussian elimination, computing inverses of matrices, and performing matrix operations.

Caton and Grossman (1980) state that progressing from textbook problems to realistic applications in linear algebra frequently results in merely increasing the dimensions of the vectors or matrices involved, while the solution algorithm remains unchanged. Computers can be used in such situations. The programs presented include solution of a system of equations by Gaussian elimination, an orthonormal basis finder, a determinant evaluator, and an eigenvalue-eigenvector finder.

Both texts state that the programs should be put on the computer system and become a part of the system's canned program library.

Blattner (1981) at California State University, Northridge uses computers for demonstrations in class, including programs that manipulate matrices. One is a matrix inversion program that works in common fractions on any 3 x 3 matrix with reasonably small entries. It provides three instructional modes of operation: the computer performs the entire inversion step-by-step; the computer performs the arithmetic, but the user tells the computer what operation to perform at each step; and the user specifies the operation at each step and also performs half the arithmetic. The computer does the arithmetic on the emerging inverse and will check steps when asked to.

San Jose State University (SJSU) has a package of programs available for linear algebra. The programs will find the determinant or inverse of a student provided n x n matrix, or multiply or add student entered matrices. A practice program helps students row reduce matrices. The program asks which operation would you like: multiply a row through by a nonzero constant; interchange two rows; or add a multiple of one row to another row. To multiply a row through by a nonzero constant, the program asks which row, whether you want to multiply or divide, and by what number. Then the operation is performed left to right and changes the color of the entries in the row. To interchange two rows, the program asks which rows and performs the interchange. To add a multiple of one row to another row,

the program asks which row will be changed?, which row will be used to change it?, and by what number will the row that will be used to change the specified row first be multiplied? Then the indicated operation is performed left to right, changing the color of the numbers in the row to a new color. The program gives decimal approximations to fractions, rather than a reduced fraction. (Mah, 1981).

In the SJSU package a graphics demonstration program allows students to enter the vertices of a triangle and choose transformations in the plane. The program shows the first quadrant with $0<=x<=120$ and $0<=y<=100$. The user may transform the original or the last triangle displayed. Available transformations include rotation through any angle, shearing x by any factor and shearing y by any factor, dilating a triangle by any factor, and reflecting a triangle about the line y=x. Each new triangle is drawn in a color which corresponds to the transformation performed (Fitting, 1981).

The SJSU programs are public domain programs written on the Compucolor II microcomputer and are available through the San Mateo Microcomputer Center, 333 S. Main Street, Redwood City, CA 94063.

Programs which provide inverses and products of matrices or solutions to simultaneous linear equations are useful when the answer is the most important ingredient. However, they do little to enhance student understanding of

the processes involved. The SJSU practice program in row reduction, on the other hand, helps students to review the process of row reduction without being encumbered with arithmetic. While the transformation program provides a demonstration of chosen transformations, it does not orient the student toward any learning goal.

Linear algebra is an area where computers can be implemented in more interactive ways. Few practice programs, demonstrations, or discovery-oriented programs are available. While the SJSU programs are a step in beginning, only two of them do more than perform computations for the students. Programming algorithms helps students to understand the algorithms better, but programming knowledge is a necessary prerequisite.

CHAPTER 3

CHOOSING A TOPIC

When I undertook this project, I wanted to investigate how computers are used in the teaching of mathematics at the junior college level, and the types of programs that are available. After reviewing much of the literature in this area I found that computers are used in a variety of ways.

In pre-calculus courses computers are used to administer and score placement tests, for diagnostic testing, to provide individualized mathematics skills instruction, as well as to administer achievement tests at the end of a course.

In algebra and trigonometry the computer is used to assist instruction. Computers are used to present drill, to practice with hints on the principles involved, for discovery learning or reinforcement of concepts by demonstration, for tutorials involving the previous three categories, to provide opportunity for problem solving involving computations suitable for a computer, and for simulation of realistic problems and situations.

In calculus courses computers are used in a variety of

ways. They are used to display graphics as an aid to calculus and for problem solving involving computational procedures. They are also used to present tutorials and instructional modules to supplement regular calculus courses.

There are several instructional advantages in using the computer. One is the computer's unique ability to lock in and intensify instruction for each student's weaker topics. Another is the computer's ability to analyze student's answers to computational exercises during practice, and provide immediate, individualized help. The computer's computational speed can be used to allow discussion of many examples of a concept, for inclusion of topics normally omitted or reserved for a subsequent course, and for more realistic applications in the field of mathematics under study.

Based on the results of my library research, linear algebra seems to be an area where computer use is not very well represented. Implementations using computers generally involve one of the following: students program solutions to typical linear algebra exercises; the computer provides direct solutions to exercises; or the computer demonstrates matrix manipulations and transformations.

In the first technique a student knowledge of programming is essential. While the number of students who can program is increasing, there remain many students who

cannot write programs. If the teacher writes lessons in which the learning takes place during programming, then this places a limit on the kind and number of students who can benefit from the lesson. In order to involve all students in exercises using the computer, the teacher could write programs which provide for learning or reinforcement in the execution rather than in the writing of the programs.

Programs which provide direct answers to the student do little to enhance the student's understanding of the processes involved.

In the area of linear algebra there are few programs available for practice, demonstration or discovery. Practice on the ideas in linear algebra without the encumbrance of arithmetic may help to reinforce many of the concepts. A program of this type is the row reduction program used at San Jose State University (SJSU). Another program that could help reinforce the concepts would be a practice program on matrix addition, matrix multiplication, or scalar multiplication. Since the computer can perform computations quickly, students can concentrate on the processes and concepts involved.

Demonstrations and discovery-oriented programs could also help in learning linear algebra. The transformation program used at SJSU is strictly a graphics demostration program. The program asks the user to enter the vertices of a triangle and choose transformations in the plane, then it

performs the transformations. This program could be modified to be a problem-solving program which also challenges the student instead of merely a demonstration of results.

Another possible program would be a tutorial on the Gram-Schmidt orthonormalization process, with graphics displays to show the process pictorially.

Programs for practice, demonstrations, and discovery can help in learning linear algebra. Since there are few programs available in this area, I have chosen to create a sample program to provide demonstration of linear transformations on the plane in the form of a challenge for the user.

CHAPTER 4

DOCUMENTATION OF PROGRAM

Section 1. Principles on which the Program was Based

Because less effort is required to produce reasonable graphics and text combinations, I chose to write my program in BASIC for the Compucolor II. Different foreground and background colors can be chosen which makes it possible to put many graphs on the same display. Text can be written in two sizes of type and can be placed anywhere on the screen along with the graphics. This allows questions to be asked while the graphics are displayed. 32 lines of 64 characters each make it possible to spread out information for easy reading. The availability of this computer, as well as large scale graphics (screen resolution of 127 x 127 blocks in eight colors) to use for demonstration in class also influenced my decision. There is some linear algebra material already in existence for use on the Compucolor II, so an instructor using these programs could add my program to their collection.

According to the text, MATHEMATICS TEACHING, (Travers, et. al., 1977) mathematical learning can be divided into three major goal classifications: knowledge, understanding, and problem solving.

Knowledge involves the learning of facts and skills. Facts are the specific statements the students are to recognize or recall. Skills, on the other hand, are the procedures in which the student is to become proficient. Knowledge is obtained through a certain degree of repetition or practice in the learning process. This should include variations, and may involve visual stimulus, games devised to practice skills, novel problems that require the knowledge or skill to be learned, as well as changing the pace in the learning environment by varying individual work, group activities, and games.

The term understanding is defined as a level of learning a general statement which enables a student to apply it in problem situations. To demonstrate that a student has acquired an understanding, the student would not simply state it. Students would show that they could apply the understanding in a particular situation. The distirguishing characteristic of understandings is that they are to be applied, derived, or used to deduce a consequence. Given an exercise which can be solved with the aid of an understanding, the student should be able to use the acquired understanding and perhaps some items of knowledge and some skills to solve the exercise.

A generalization or concept may be taught as an item of knowledge, as a skill, or as an understanding. If it is taught as an item of knowledge, the goal for the student is

to be able to recall and state it. If it is taught as a skill, the goal is for the student to use it to solve routine exercises. If it is taught as an understanding, the goal is for the student to apply it, derive it, or use it to deduce a consequence.

The goals in the classification of problem solving emphasize the processes of mathematics. The student must do something or think something. Goals in the other classifications are not process oriented in the sense that the student selects the process. To demonstrate that a student has acquired knowledge, the student recalls a fact, to demonstrate that a student understands a statement, the student applies it. In such an instance, the student may use one or more mathematical processes; but the nature of the exercise is such that a particular generalization, principle, theorem, definition, or axiom is applicable. The student would typically select the appropriate understanding and apply it. Problems here are routine, the student would employ a mathematical process determined by the exercise.

Items designed for students to demonstrate that they have achieved goals in the problem solving classification will be solved by mathematical processes. Typically, two major characteristics of these items are that usually more than one process will be needed, and there will be more than one avenue for the student to follow to solve the

non-routine problem.

Development of my program on Linear Transformations on the Plane considered this learning hierarchy. The program involves knowledge of the definitions of rotation, dilation, reflection, x-axis, y-axis, line graphs, and triangles. Students must understand the effects of rotations, dilations, and reflections; the graphs of the equations $y=x$ and $y=-x$; and the rigidity of a triangle. Problem solving is involved since the student must choose which transformation produces the desired result. Three levels of difficulty each provide repetition and variation since transformations are randomly chosen. Depending on the difficulty level selected the exercise presented may include one, two, or three transformations. Hence, more than one process may be involved and the order of application of the transformations allows the student more than one avenue of solution.

Discovery oriented programs involve activities using an inductive approach to learning. That is, presenting problems which the student solves through trial and error. It is the opposite of rote or drill learning. The aim of the discovery approach is the deeper understanding that results from grappling with a puzzling problem (Heinich, 1982).

I feel the program presents non-routine problems in a self-correcting, discovery oriented manner.

Section 2. Overview of Program

LINEAR TRANSFORMATIONS ON THE PLANE

This program provides demonstration of linear transformations on the plane in the form of a challenge for the user.

It draws a white triangle and a light blue (cyan) triangle on coordinate axes representing all four quadrants. The user is asked to determine which transformation(s): rotation; dilation; and/or reflection will change the white triangle into the light blue triangle.

The user has a choice of three difficulty levels which will incorporate one, two, or three transformations into the challenge presented. The program randomly generates:

(1) rotations in 30 degree increments from 30 to 330 degrees;

(2) dilation factors of 0.25, 0.3, 0.5, 2, and 3; and

(3) reflections across the x-axis, y-axis, line y=x, and the line y=-x.

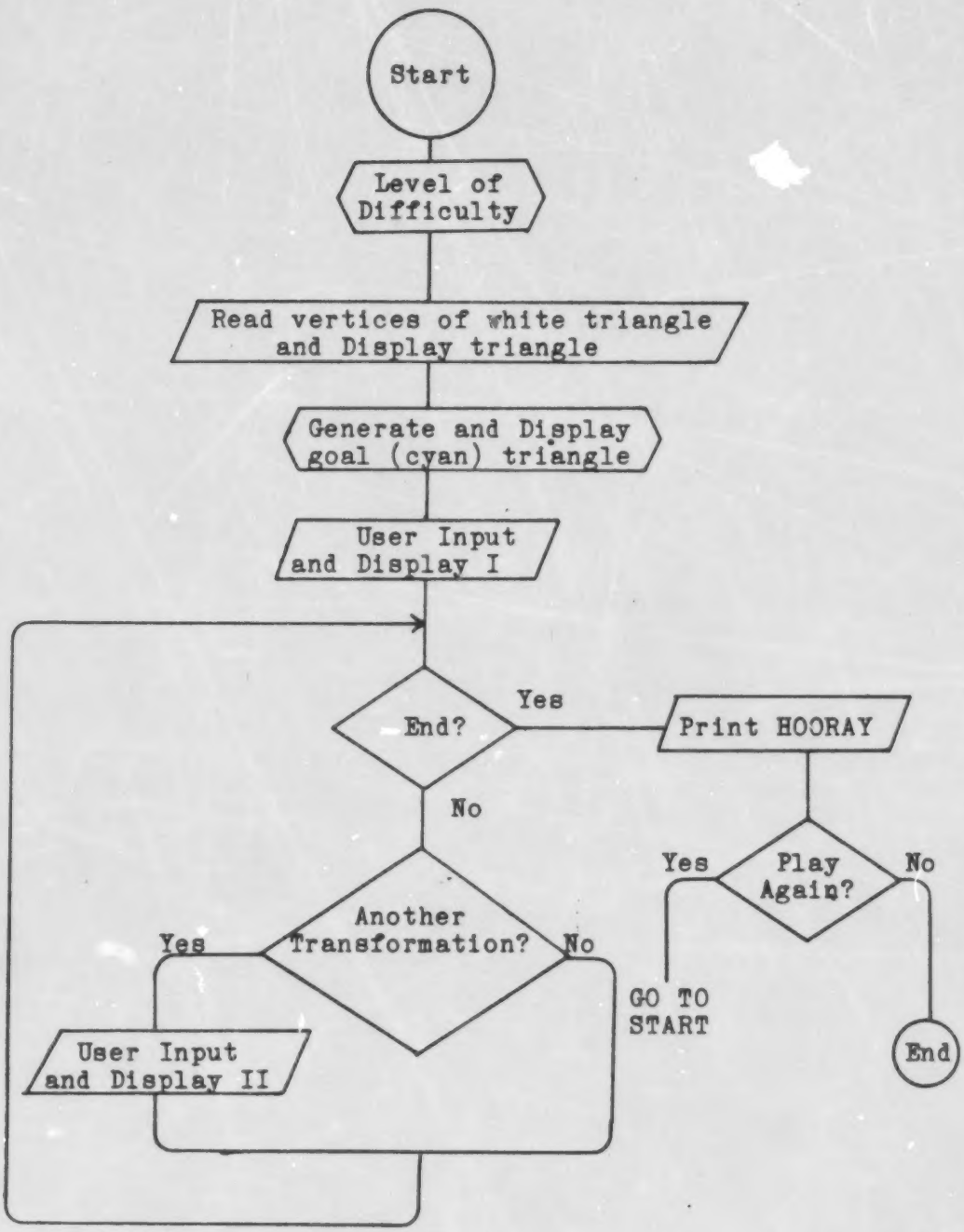
User chosen ROTATIONS are drawn in (royal) blue, DILATIONS are drawn in red, and REFLECTIONS are drawn in green. The program offers the choice of starting with the

white and light blue triangles again before each new transformation is chosen by the user. This provides an opportunity to clear the screen if it has become cluttered.

When the triangle generated by the user agrees with the light blue triangle, the program prints "HOORAY!" and asks if the user would like to play again.

Section 3. Modular Flowchart

LINEAR TRANSFORMATIONS ON THE PLANE



WHICH LEVEL OF DIFFICULTY WOULD YOU LIKE?

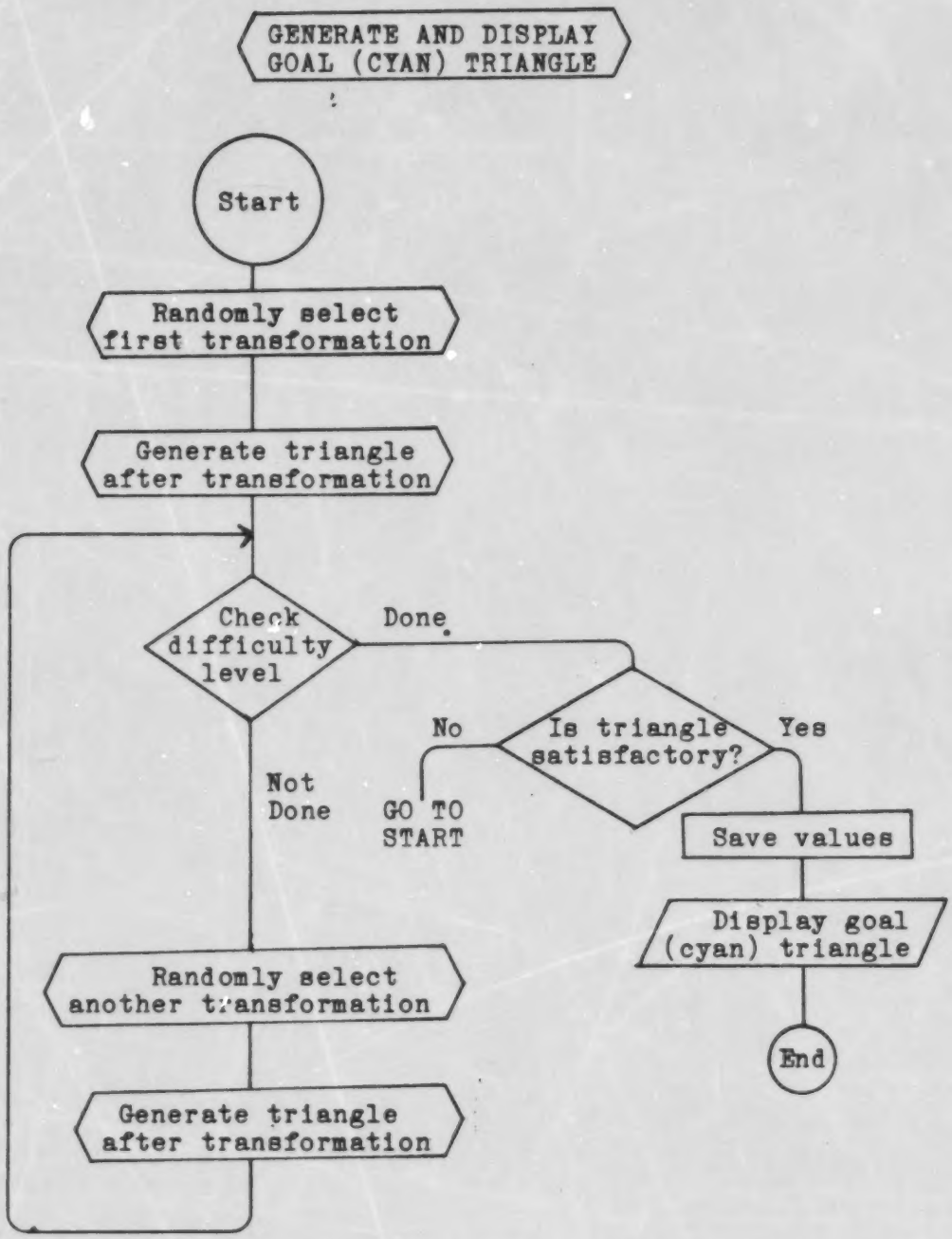
(1) EASY

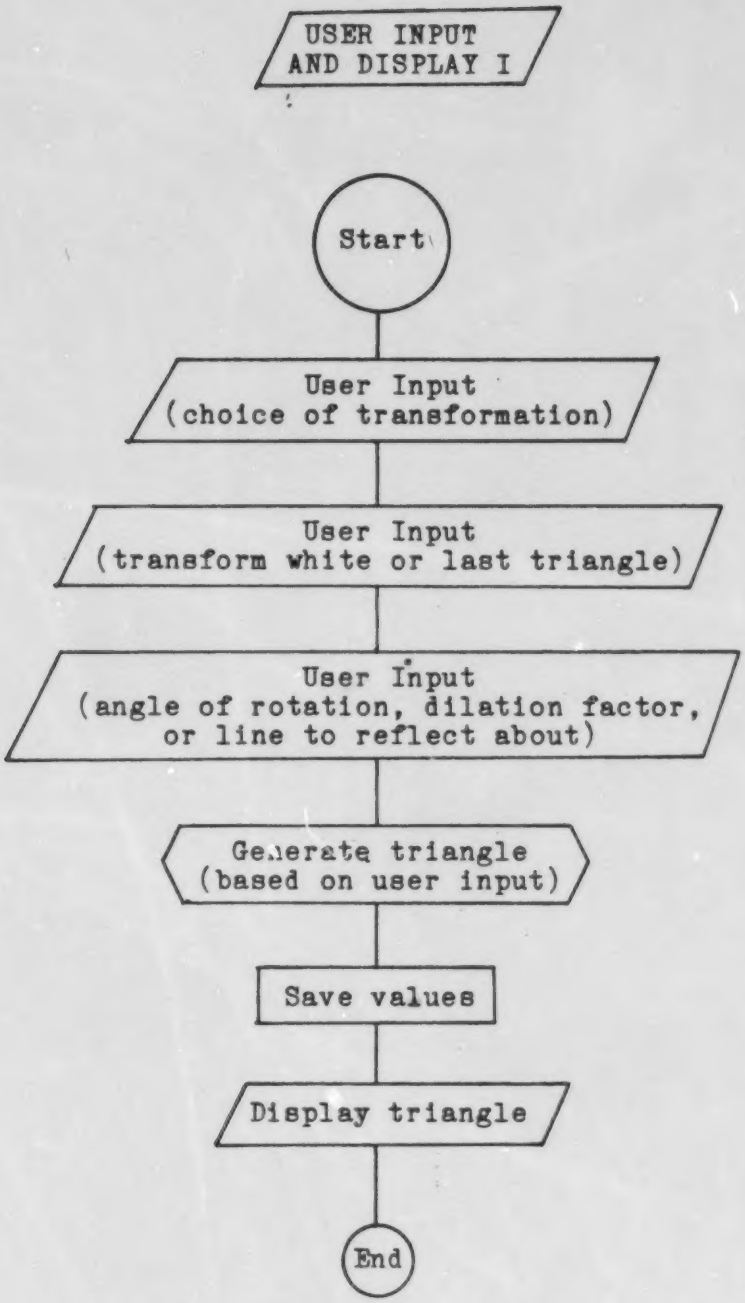
(2) MEDIUM

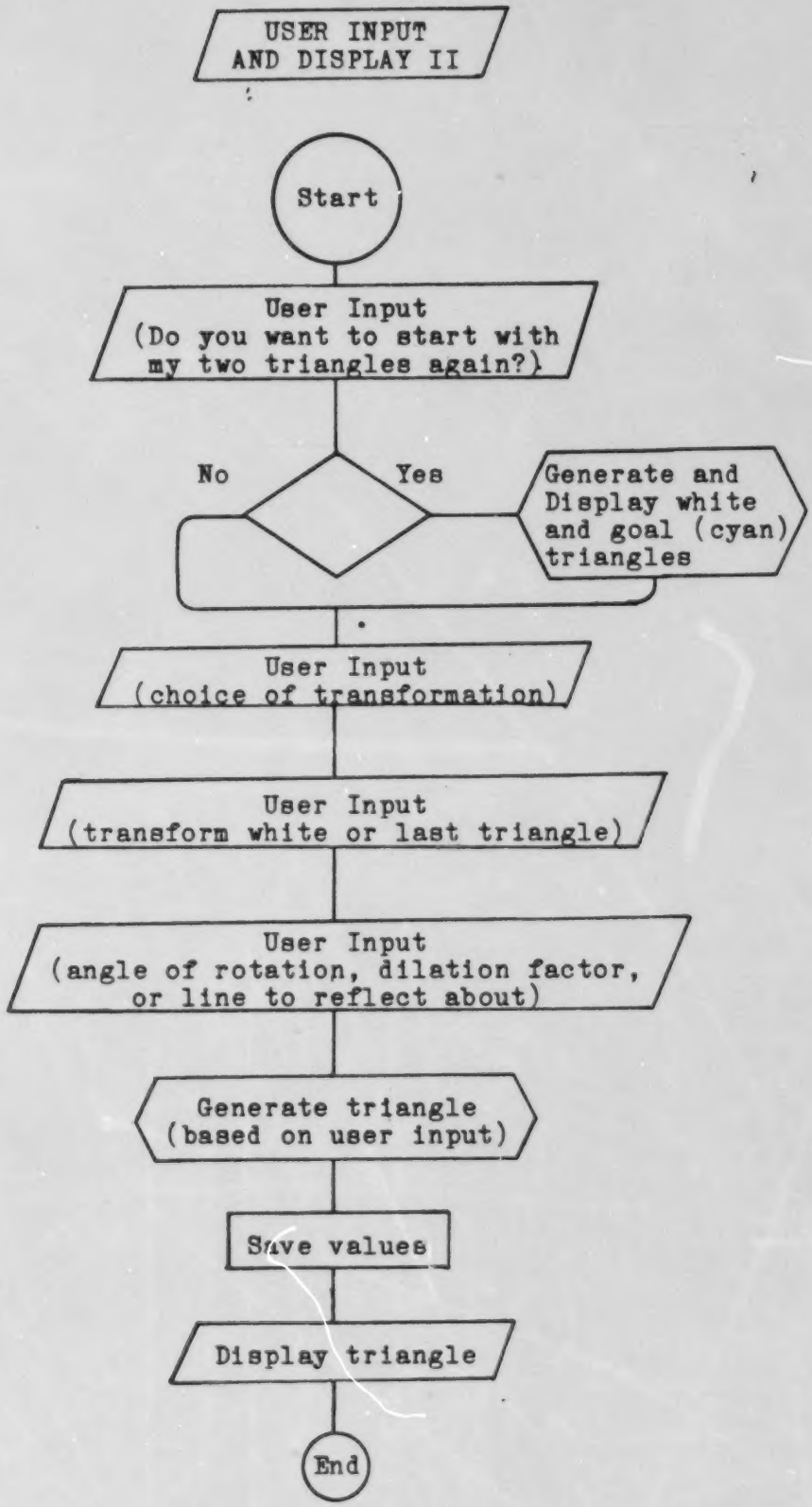
(3) HARD

TYPE THE NUMERAL OF YOUR CHOICE.

The input here sets a flag to 1, 2, or 3 which is utilized in the routine for GENERATE AND DISPLAY GOAL TRIANGLE.


GENERATE AND DISPLAY GOAL (CYAN) TRIANGLE
Start
Randomly select first transformation
Generate triangle after transformation
Check difficulty level
Done
Not Done
Is triangle satisfactory?
No
GO TO START
Yes
Save values
Display goal (cyan) triangle
End
Randomly select another transformation
Generate triangle after transformation


USER INPUT
AND DISPLAY I
Start
User Input
(choice of transformation)
User Input
(transform white or last triangle)
User Input
(angle of rotation, dilation factor,
or line to reflect about)
Generate triangle
(based on user input)
Save values
Display triangle
End


USER INPUT
AND DISPLAY II
Start
User Input
(Do you want to start with
my two triangles again?)
No
Yes
Generate and
Display white
and goal (cyan)
triangles
User Input
(choice of transformation)
User Input
(transform white or last triangle)
User Input
(angle of rotation, dilation factor,
or line to reflect about)
Generate triangle
(based on user input)
Save values
Display triangle
End

Section 4. Specific Instructions to the User

This program was written for a 16K Compucolor Computer.

Preliminaries:

The keyboard is similar to a typewriter except that the letters are in capitals, so the shift key is not needed to make capitals.

1. You must use the "[ø]" key for the number zero as the computer interprets the number "O" and the letter "O" differently.

2. You must use the "[! 1]" key for the number one.

3. The "[= -]" key will be used to make a negative sign.

Note: You may use the numbers and negative sign in the lower right hand corner of the keyboard instead of the keys specified in parts 1, 2, and 3 above.

4. Whenever you answer a question or enter a number you must press the RETURN key afterwards.

5. If you make a typing error press the "[←——]" key to back up over it, make your correction, then press the RETURN key.

To Run the Program:

1. Locate the DISK DRIVE which is a box with a black door.

2. Open the door on the disk drive. Put the disk LINEAR TRANSFORMATIONS ON THE PLANE in the disk drive inserting it with the oval hole (through which you can see the surface of the disk) away from you and the label up. Close the door on the disk drive.

3. Lock the CAPS LOCK key (at the left end of the space bar) in the down position.

4. Press the CPU RESET key which is a red key in the upper right hand corner of the keyboard.

5. Press the AUTO key which is a brown key in the upper left hand corner of the keyboard.

The program is now running.

Examples and Explanations:

The purpose of this program is to provide demonstrations of linear transformations on the plane in the form of a challenge for you, the user. It draws a white triangle and a light blue (cyan) triangle on coordinate axes representing all four quadrants. Your job is to determine which transformation(s): rotation, dilation, and/or reflection will change the WHITE triangle into the LIGHT BLUE triangle. Hence, the computer says:

```
TRY TO MAKE THE WHITE TRIANGLE FALL ON THE LIGHT BLUE
TRIANGLE BY CHOOSING APPROPRIATE TRANSFORMATIONS.
```

Next it says:

```
WHICH LEVEL OF DIFFICULTY WOULD YOU LIKE?
     (1) EASY
     (2) MEDIUM
     (3) HARD
TYPE THE NUMERAL OF YOUR CHOICE.
```

You will type the number 1, 2, or 3 specifying the difficulty level you would like.

The first difficulty level, (1) EASY, means the program will generate a light blue triangle for you with ONE transformation involved to get from the WHITE triangle to the LIGHT BLUE triangle.

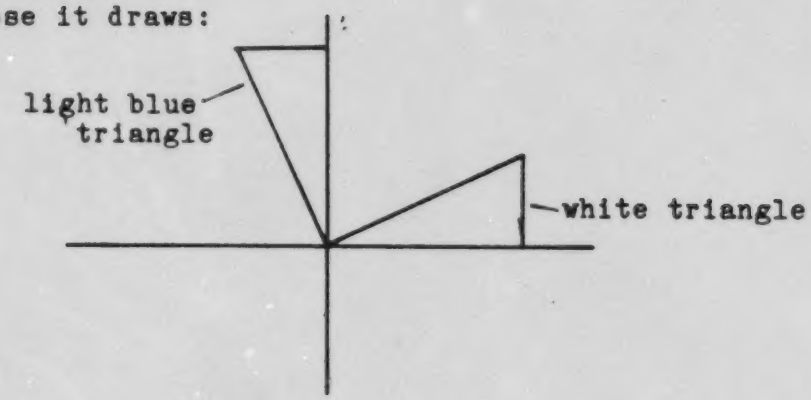
In the second level of difficulty, (2) MEDIUM, the program will generate a light blue triangle with TWO different transformations involved to get from the WHITE triangle to the LIGHT BLUE triangle.

The third difficulty level, (3) HARD, will generate a light blue triangle with THREE different transformations involved to get from the WHITE triangle to the LIGHT BLUE triangle.

Let's pick the first difficulty level, so we'll type the number "1" and press the RETURN key.

The program will draw axes, and a white triangle, then generate and draw a light blue triangle with one transformation incorporated to get FROM the WHITE triangle TO the LIGHT BLUE triangle.

Suppose it draws:



for us, and next asks:

WHICH TRANSFORMATION WOULD YOU LIKE TO PERFORM?
(1) ROTATION; (2) DILATION; (3) REFLECTION; (4) END.

The program randomly generates:

(1) ROTATIONS in 30 degree increments from 30 to 330 degrees;
(2) DILATION factors of 0.25, 0.3, 0.5, 2, and 3; and
(3) REFLECTIONS across the x-axis, y-axis, line y=x, and the line y=-x,

so it is to your advantage to select from these values when you guess what the program has generated for you.

In our example, we think the transformation is a rotation, so we type the number "1" for ROTATION and press the RETURN key.

The program next asks:

ROTATE THE WHITE (W) TRIANGLE OR THE LAST (L) TRIANGLE?

We have no LAST triangle yet, so we'll type the letter "W" for the WHITE triangle and press the RETURN key.

The program then asks:

THROUGH WHICH DEGREE ANGLE WOULD YOU LIKE TO ROTATE THE FIGURE?

Of course it means what angle (in degrees) would we like to rotate the WHITE triangle through, since that's what we just asked for.

Let's try 90 degrees, so we'll type the number "90" and press the RETURN key.

The program will draw the white triangle rotated 90 degrees, in royal blue. It draws all user chosen ROTATIONS in ROYAL BLUE.

If your choice agrees with the light blue triangle the program says "HOORAY!" and asks:

DO YOU WANT TO PLAY AGAIN?

Let's type the word "YES" and press the RETURN key.

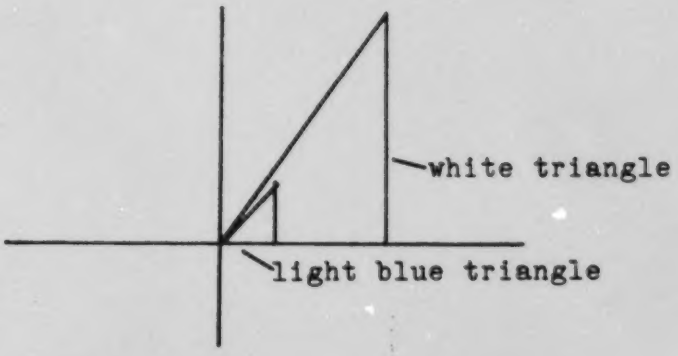
We see:

TRY TO MAKE THE WHITE TRIANGLE FALL ON THE LIGHT BLUE TRIANGLE BY CHOOSING APPROPRIATE TRANSFORMATIONS.

WHICH LEVEL OF DIFFICULTY WOULD YOU LIKE?
(1) EASY
(2) MEDIUM
(3) HARD
TYPE THE NUMERAL OF YOUR CHOICE.

Let's type the number "1" again for the first difficulty level and press the RETURN key.

Suppose it draws:



and asks:

WHICH TRANSFORMATION WOULD YOU LIKE TO PERFORM?
(1) ROTATION; (2) DILATION; (3) REFLECTION; (4) END.

Let's type the number "2" for DILATION, since the light blue triangle is a different size than the white triangle and press the RETURN key.

The program asks:

DILATE THE WHITE (W) TRIANGLE OR THE LAST (L) TRIANGLE?

We'll type the letter "W" for the WHITE triangle, as before, and press the RETURN key.

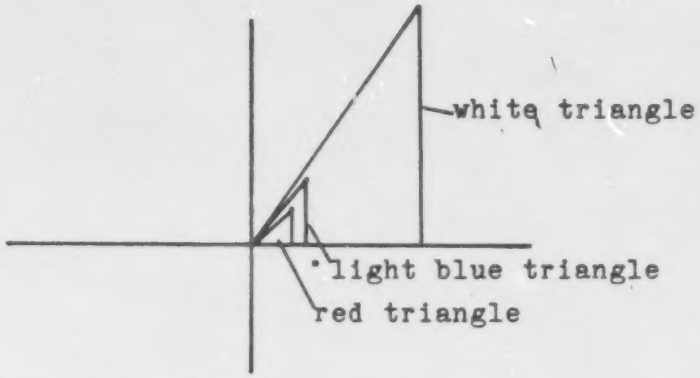
Next we are asked:

BY WHAT FACTOR DO YOU WISH TO DILATE THE TRIANGLE?

Recalling that the program generates dilation factors of 1/4, 1/3, 1/2, 2, and 3, which we must enter as .25, .3, .5, 2, and 3, respectively, let's type the number ".25" and press the RETURN key.

The program will draw the white triangle dilated by a factor of .25 in red. It draws all user chosen DILATIONS in RED.

We will see:



and the computer will ask:

DO YOU WANT TO CHOOSE ANOTHER TRANSFORMATION?

On closer investigation we notice that our dilation factor of .25 made the triangle too small, and that the side of the LIGHT BLUE triangle along the x-axis as compared to the side of the WHITE triangle along the x-axis seem to be in the ratio 1:3. So we want to try a refined DILATION factor of 1/3 or .3 on the WHITE triangle.

Let's type the word "YES" and press the RETURN key to choose another transformation.

Next the computer asks:

DO YOU WANT TO START WITH MY TWO TRIANGLES AGAIN?

This option allows us to clear the screen if it has become cluttered.

Let's type the word "YES" and press the RETURN key.

The program will then draw the axes, the white triangle and the light blue triangle again for us, then print:

WHICH TRANSFORMATION WOULD YOU LIKE TO PERFORM?
(1) ROTATION; (2) DILATION; (3) REFLECTION; (4) END.

We'll type the number "2" for DILATION and press the RETURN key.

The program asks:

DILATE THE WHITE (W) TRIANGLE OR THE LAST(L) TRIANGLE?

We'll type the letter "W" for the WHITE triangle and press the RETURN key.

The next question is:

BY WHAT FACTOR DO YOU WISH TO DILATE THE TRIANGLE?

We'll type the number ".3" and press the RETURN key.

The program draws our dilated triangle in red, prints "HOORAY!" since it agrees with the light blue triangle and asks:

DO YOU WANT TO PLAY AGAIN?

Let's type the word "YES" and press the RETURN key.

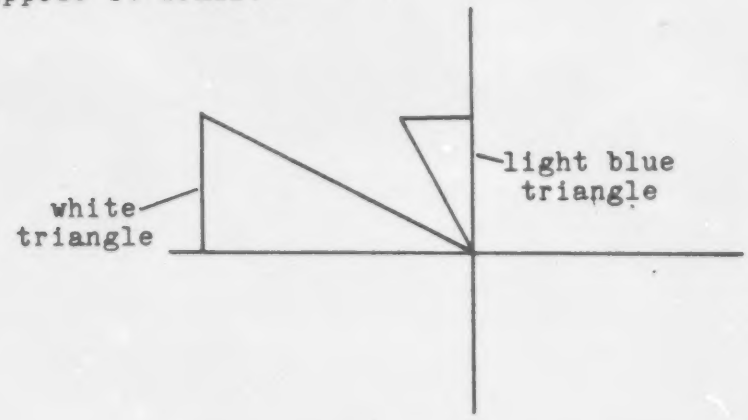
Again we see:

TRY TO MAKE THE WHITE TRIANGLE FALL ON THE LIGHT BLUE TRIANGLE BY CHOOSING APPROPRIATE TRANSFORMATIONS.

WHICH LEVEL OF DIFFICULTY WOULD YOU LIKE?
(1) EASY
(2) MEDIUM
(3) HARD
TYPE THE NUMERAL OF YOUR CHOICE.

Let's type the number "2" this time for the second level of difficulty which will incorporate two transformations into the challenge presented, and press the RETURN key.

Suppose it draws:



and asks:

WHICH TRANSFORMATION WOULD YOU LIKE TO PERFORM?
(1) ROTATION; (2) DILATION; (3) REFLECTION; (4) END.

Let's type the number "2" for DILATION and press the RETURN key.

The program asks:

DILATE THE WHITE (W) OR LAST (L) TRIANGLE?

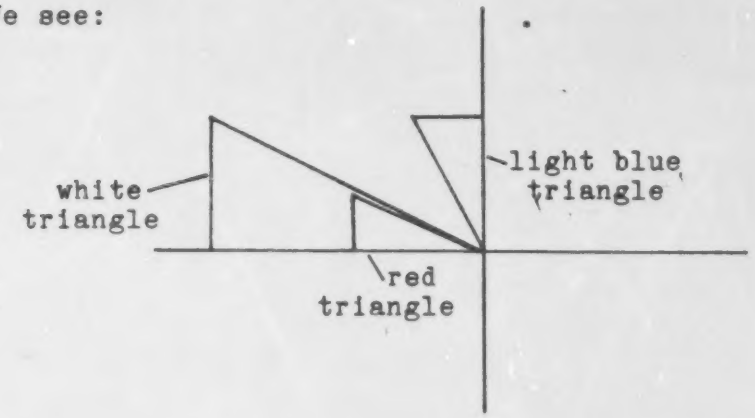
We'll type the letter "W" for the WHITE triangle and press the RETURN key.

The program asks:

BY WHAT FACTOR DO YOU WISH TO DILATE THE TRIANGLE?

Let's type the number ".5" and press the RETURN key.

We see:



and are asked:

DO YOU WANT TO CHOOSE ANOTHER TRANSFORMATION?

Let's type the word "YES" and press the RETURN key.

Next the program asks:

DO YOU WANT TO START WITH MY TWO TRIANGLES AGAIN?

This time we'll type the word "NO" since we need to transform the red triangle to get to the light blue triangle, and press the RETURN key.

We are next asked:

WHICH TRANSFORMATION WOULD YOU LIKE TO PERFORM?
(1) ROTATION; (2) DILATION; (3) REFLECTION; (4) END.

The transformation we'd like is not a rotation, since the triangle has been "flipped over" so to speak, so let's type the number "3" for REFLECTION and press the RETURN key.

We are asked:

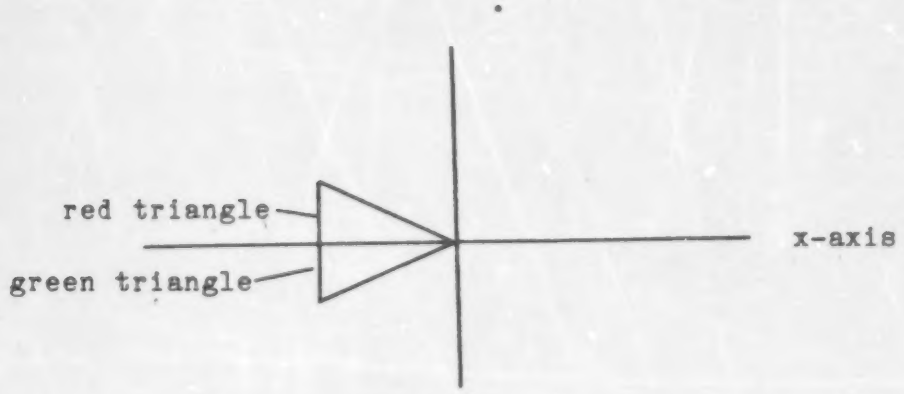
REFLECT THE WHITE (W) TRIANGLE OR THE LAST(L) TRIANGLE?

We want to reflect the red triangle which is the LAST triangle drawn, so we'll type the letter "L" for the LAST triangle and press the RETURN key.

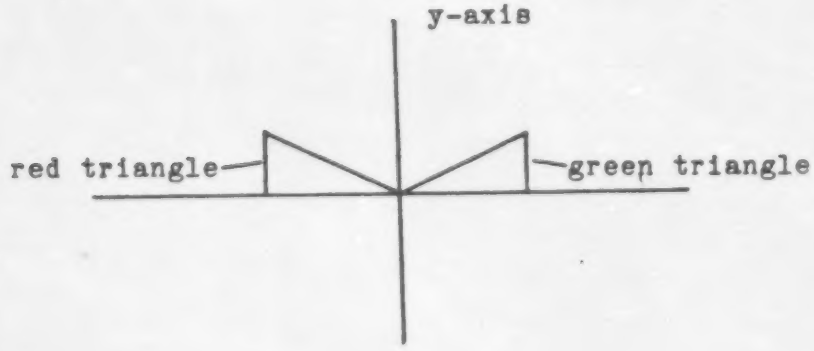
Next the program asks:

REFLECT ABOUT THE (1) X-AXIS; (2) Y-AXIS, (3) LINE Y=X;
(4) LINE Y=-X?
TYPE THE NUMERAL OF YOUR CHOICE.

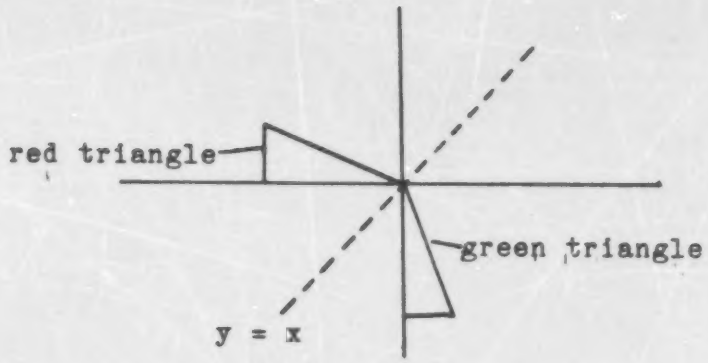
Does it look as if the red triangle was reflected about the x-axis to get to the light blue triangle? That transformation would look like:



Does it look as if the red triangle was reflected about the y-axis to get to the light blue triangle? That transformation would look like:

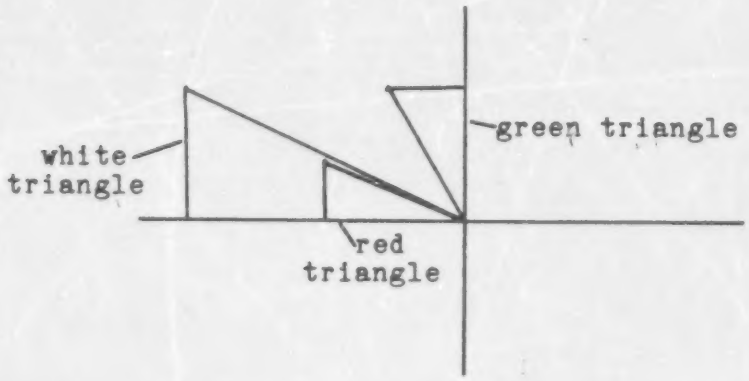


Does it look as if the red triangle was reflected about the line y=x to get to the light blue triangle? That transformation would look like:



Well, does it look as if the red triangle was reflected about the line y=-x to get to the light blue triangle?

If we type the number "4" for this choice and press the RETURN key, the program will draw the red triangle reflected about the line y-=x in green. It draws all user chosen REFLECTIONS in GREEN. We will see:



and the computer will print "HOORAY!" then ask:

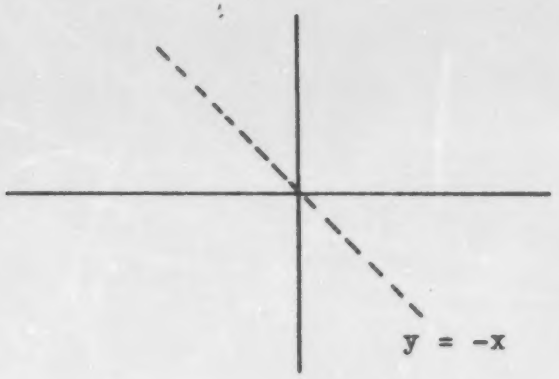
DO YOU WANT TO PLAY AGAIN?

You have a choice of typing the word "YES" to play again or the word "NO" and pressing the RETURN key to end the game.

You also may type the number "4" and press the RETURN key to end when you are asked:

WHICH TRANSFORMATION WOULD YOU LIKE TO PERFORM?
(1) ROTATION; (2) DILATION; (3) REFLECTION; (4) END.

Incidentally, the line y=-x which we reflected about in the last example looks like:



Have fun playing with and learning about linear transformations on the plane!

Mathematical Concepts

The transformations of dilation, reflection, and rotation are all performed in this program through matrix multiplication.

Labelling the vertices of the triangle (X1,Y1), (X2,Y2), and (X3,Y3), the matrix representing these vertices is set up in the form $\begin{bmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \end{bmatrix}$.

Dilations are performed through matrix multiplication. Given a dilation factor, F1, we can multiply the matrix of vertices by this constant, that is,

$$F1* \begin{bmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \end{bmatrix},$$

which gives the matrix $\begin{bmatrix} F1*X1 & F1*X2 & F1*X3 \\ F1*Y1 & F1*Y2 & F1*Y3 \end{bmatrix}$ of dilated vertices of the new triangle.

Reflections are obtained through matrix multiplication.

To reflect about the x-axis the matrix of vertices is multiplied by the matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, that is,

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} * \begin{bmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \end{bmatrix} = \begin{bmatrix} X1 & X2 & X3 \\ -Y1 & -Y2 & -Y3 \end{bmatrix}.$$

This leaves the x-coordinates as they are and negates the y-coordinates of the triangle, thus producing a new matrix of vertices which reflects the triangle about the x-axis.

To reflect about the y-axis the matrix of vertices is multiplied by the matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix},$$

that is,

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} * \begin{bmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \end{bmatrix} = \begin{bmatrix} -X1 & -X2 & -X3 \\ Y1 & Y2 & Y3 \end{bmatrix}.$$

This leaves the y-coordinates alone and negates the x-coordinates of the triangle, producing a new matrix of vertices which reflects the triangle about the y-axis.

To reflect about the line y=x, the x and y-coordinates of the vertices of the triangle must be exchanged. Multiplying the matrix of vertices by the matrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

that is,

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \end{bmatrix} = \begin{bmatrix} Y1 & Y2 & Y3 \\ X1 & X2 & X3 \end{bmatrix}$$

produces a new matrix of vertices of a triangle with the x and y-coordinates exchanged, which reflects the triangle about the line y=x.

To reflect about the line y=-x, the x-coordinates must be replaced by the opposite of the y-coordinates and the

y-coordinates must be replaced by the opposite of the x-coordinates. Multiplying the matrix of vertices by the matrix

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix},$$

that is,

$$\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} * \begin{bmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \end{bmatrix} = \begin{bmatrix} -Y1 & -Y2 & -Y3 \\ -X1 & -X2 & -X3 \end{bmatrix}$$

produces a new matrix of vertices of a triangle which reflects the triangle about the line y=-x.

Rotation through an angle θ is performed through multiplying the matrix of vertices by the matrix

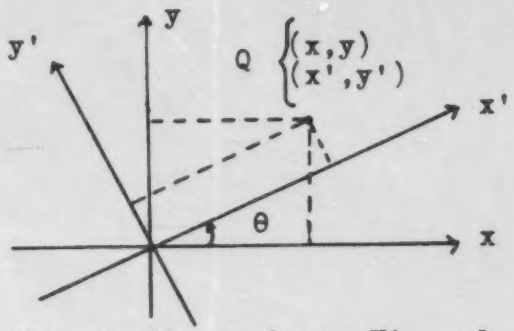
$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

that is,

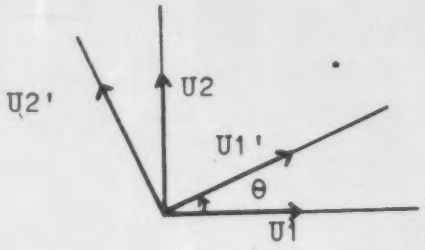
$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} * \begin{bmatrix} X1 & X2 & X3 \\ Y1 & Y2 & Y3 \end{bmatrix}$$

will give a resulting 2 x 3 matrix of new vertices which rotates the triangle through an angle θ.

To derive this matrix, given a rectangular xy-coordinate system a new x'y'-coordinate system is obtained by rotating the xy-system counterclockwise about the origin through an angle θ. When this is done each point Q in the plane has two sets of coordinates: coordinates (x,y) relative to the xy-system and coordinates (x',y') relative to the x'y'-system.



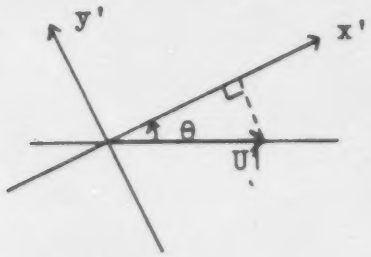
By introducing unit vectors U1 and U2 along the positive x and y-axes and unit vectors U1' and U2' along the positive x' and y' axes, we can regard this rotation as a change from an old basis B={U1,U2} to a new basis B'={U1',U2'}.



Thus the new coordinates (x',y') and the old coordinates (x,y) of a point Q will be related by

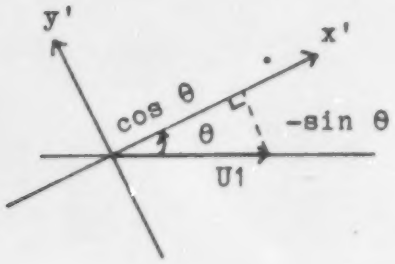
Equation (I) $$\begin{bmatrix} x' \\ y' \end{bmatrix} = P^* \begin{bmatrix} x \\ y \end{bmatrix}$$

where P is the transition matrix from B to B'. To find P we must determine the coordinate matrices of the old basis vectors U1 and U2 relative to the new basis. To find the components of U1 in the new basis consider the figure:



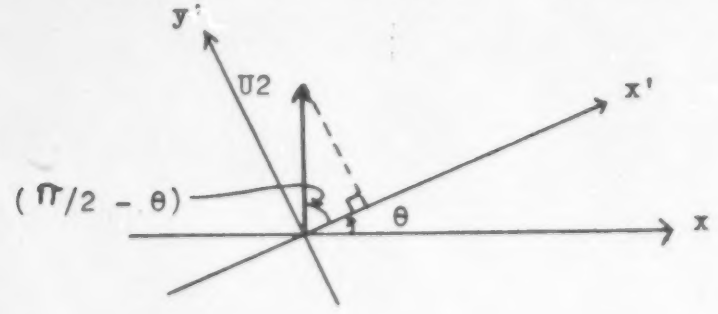
The component of U1 in the x' direction is cos θ since the hypotenuse of the right triangle is a unit vector of length one, and cos θ = side adjacent/hypotenuse, that is cos θ = side adjacent/1. So, the length of the side adjacent (the component in the x' direction) is cos θ.

We can find the component in the y' direction, which is -sin θ, by applying the Pythagorean Theorem, the identity sin^2 θ + cos^2 θ =1, and noting that the sine function is negative in the fourth quadrant (with respect to the x'y'-system).



Hence, the components of U1 in the new basis are cos θ and -sin θ so that $[U1]_{B'} = \begin{bmatrix} \cos\theta \\ -\sin\theta \end{bmatrix}$

To find the components of U2 in the new basis consider the figure:



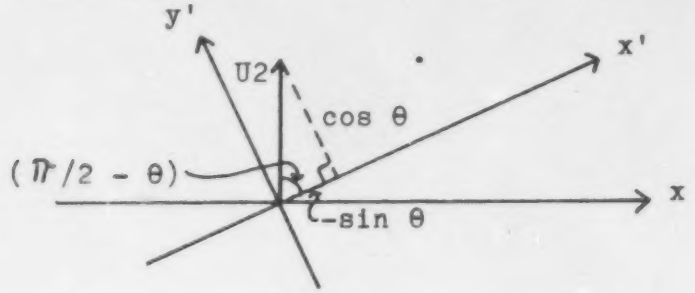
The component of U2 in the x' direction is arrived at in the

same manner as we found the x'-component of U1. Noting that U2 is the hypotenuse of the right triangle and that it is of length one,

$$\cos(\pi/2 - \theta) = \text{side adjacent/hypotenuse}$$
$$= \text{side adjacent/1.}$$

So, the component in the x' direction (the length of the side adjacent) is $\cos(\pi/2 - \theta)$ which is equal to $\sin\theta$.

We find the component of U2 in the y' direction is $\sin(\pi/2 - \theta)$ which is equal to $\cos\theta$ in a manner similar to that used to find the component of U1 in the y' direction.



Hence, the components of U2 in the new basis are $\sin\theta$ and $\cos\theta$, so that $[U2]_{B'} = \begin{bmatrix}\sin\theta\\ \cos\theta\end{bmatrix}$

Thus, the transition matrix from B to B' is

$$P = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

and (I) becomes

$$\begin{bmatrix}x'\\ y'\end{bmatrix} = \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} * \begin{bmatrix}x\\ y\end{bmatrix}$$

(Anton, 1981, pp 197-198).

CHAPTER 5

CONCLUSION

The computer program was designed to help students learn a topic in which the unique features of a computer could be used to do something that is difficult for a teacher to do alone. Already available programs include the use of drill to reinforce knowledge, practice to reinforce understanding, but involve little problem solving. I elected to write a program involving problem solving. Goal orientation was accomplished by presenting two triangles, and asking the student to solve the problem of transforming the first into the second.

The use of graphics and color made this presentation one that is not easily duplicated by other modes of instruction. Mathematical methods involve the use of matrix multiplication. Documentation was written to make the program accessible to students uninitiated in computer use.

There is still room for improvement of my program. Most commercial programs have available several commands which can be executed at any time while running the programs. One of these features could be "Help" which gives

hints about what to look for. A second feature could be the choice of going "Back" to the previous frame. This would entail storing all the information of the preceeding graphics display to be able to reproduce the previous status of the display. A third feature might be an "Escape" key to start the present sequence over with the same data, that is, without changing triangles. This is available currently in the program, but only at specific times during the execution.

It would be nice if rotations and dilations were drawn in an animated fashion. Triangles of smaller increments could be drawn and erased in succession to give the effect of motion while displaying a rotation or dilation.

These features and other items could be incorporated to give the user more control over the program.

I feel the discovery experience which this program offers is valuable. A brief orientation to the topic and to the computer may prove useful before running the computer program. The presence of the teacher or an aid to answer questions while students run the program might be helpful. Follow up materials for use in class may be important and some method of testing the student's understanding of the material in the computer program could be devised. Related subject matter which can be taught effectively in class could also be presented concurrent with the use of my program. The development of a unit of material that uses

this computer program would be a worthwhile project.

The concept of setting a goal and the idea that the student would spend some time with the program instead of merely looking at a demonstration of results are two of the features of the program.

Besides linear transformations on the plane, there are many other topics in linear algebra, that could be taught more effectively if computers were used. Practice on the ideas in linear algebra without the encumbrance of arithmetic may help to reinforce many of the concepts. A practice program on matrix addition, matrix multiplication, or scalar multiplication could help reinforce the concepts. Since the computer can perform computations quickly, students can concentrate on the processes and concepts involved. Another area where computers could be used would be a tutorial program on the Gram-Schmidt orthonormalization process, with graphics displays to show the process pictorially.

These topics could provide the basis for future work. The unique features of the computers can make many areas in the study of mathematics more accessible and meaningful than a teacher working alone without computers.

Appendices A and B (pages 80-89) are missing.

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Abbreviations Used:

American Mathematical Monthly (AMM)
Computer-Using Educators Softswap (CUE)
Dissertation Abstracts International (DAI)
National Consortium on Uses of Computers in Mathematical Sciences Education (NCCMSE)
National Council of Teachers of Mathematics (NCTM)
National Science Foundation (NSF)
Mathematical Association of Two Year Colleges (MATYC)